A NEW APPROACH FOR COMPUTING LSP PARAMETERS FROM 10TH-ORDER LPC COEFFICIENTS

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ABSTRACT

An new approach for computing Line Spectrum Pair (LSP) parameters is proposed. The LSP parameters are proved to be the roots of two 5-degree algebra equations. The root-solving procedure of such an equation is divided into three sections: 1) compute the inflexions and split (-2,2) into 5 subintervals; 2)search three roots in the shortest three subintervals respectively; 3) calculate the other two roots by formulas.

1. INTRODUCTION

LSP parameters are widely applied in speech coding [1], speech recognition [2] and other domains. Many methods have been developed for computing LSP parameters [3]. Some methods concerned the LSP parameters in θ-domain, others in x-domain. The emphasis of this paper is on decreasing the computational complexity for 10th-order LSP parameters in x-domain. The properties of symmetrical polynomials are used and an new approach for calculating the LSP parameters is proposed. Comparing with the method in [3], the computational procedure is effective.

2. DEFINITION OF LSP PARAMETERS

Given an input speech sequence $\{x(n)\}$, the LPC inverse filter is defined by

$$A_p(z) = 1 + \sum_{k=1}^{p} a_k \cdot z^{-k}.$$

Soong and Juang [1] proved that the roots of the following two symmetrical polynomials

$$\begin{aligned}\hat{P}_p(z) &= A_p(z) + z^{-(p+1)}A_p(z^{-1})\\ \hat{Q}_p(z) &= A_p(z) - z^{-(p+1)}A_p(z^{-1})\end{aligned}$$

interlace on the unit circle when p is even. $\hat{P}_p$ has a zero at $z=-1$, while $\hat{Q}_p$ has a zero at $z=1$. The roots of

$$\begin{aligned}P_p(z) &= \frac{\hat{P}_p(z)}{(1+z^{-1})}\\ &= \prod_{k=1}^{p/2}(1+d_k^{(P)}z^{-1}+z^{-2}) \qquad (1)\end{aligned}$$

$$\begin{aligned}Q_p(z) &= \frac{\hat{Q}_p(z)}{(1-z^{-1})}\\ &= \prod_{k=1}^{p/2}(1+d_k^{(Q)}z^{-1}+z^{-2}) \qquad (2)\end{aligned}$$

are complex conjugated and their angles (upper semicircle of the z-plane only) are called the LSP parameters (in θ-domain) and denoted by $\theta_k^{(P)}$, $\theta_k^{(Q)}$, $k = 1,2,\cdots,p/2$. The LSP parameters in x-domain are defined as $-\frac{1}{2}d_k^{(P)} = \cos\theta_k^{(P)}$ and $-\frac{1}{2}d_k^{(Q)} = \cos\theta_k^{(Q)}$, $k=1,2,\cdots,p/2$.

3. ALGEBRA DECOMPOSITION

Let $p=10$ in this paper. The properties of the roots of (1) ensure $P_{10}(z)$ to be symmetrical polynomials:

$$P_{10}(z) = 1 + \left(\sum_{k=1}^{9} b_k z^{-k}\right) + z^{-10}, \qquad (3)$$

where

$$\begin{cases} b_1 = b_9 = a_1 + a_{10} - 1,\\ b_2 = b_8 = a_2 + a_9 - b_1,\\ b_3 = b_7 = a_3 + a_8 - b_2,\\ b_4 = b_6 = a_4 + a_7 - b_3,\\ b_5 = a_5 + a_6 - b_4. \end{cases} \qquad (4)$$

Moreover, according to Algebra Basic Theorem, $P_{10}(z)$ can be decomposed as

$$(1+d_1^{(P)}z^{-1}+z^{-2})\left[1+\left(\sum_{k=1}^{7}e_kz^{-k}\right)+z^{-8}\right], \quad (5)$$

where $e_1=e_7$, $e_2=e_6$, $e_3=e_5$. Expand the right member of (5) and compare the coefficients, the following equations can be obtained:

$$\begin{cases} e_1+d_1^{(P)}=b_1, \\ e_2+d_1^{(P)}e_1+1=b_2, \\ e_3+d_1^{(P)}e_2+e_1=b_3, \\ e_4+d_1^{(P)}e_3+e_2=b_4, \\ 2e_3+d_1^{(P)}e_4=b_5. \end{cases} \quad (6)$$

From (6), $d_1^{(P)}$ is the root of

$$\begin{aligned} &x^5-b_1x^4+(b_2-5)x^3+(4b_1-b_3)x^2 \\ &+(5-3b_2+b_4)x+(2b_3-2b_1-b_5)=0. \end{aligned} \quad (7)$$

Similarly, $d_2^{(P)}\sim d_5^{(P)}$ are also the roots of (7), and $d_1^{(Q)}\sim d_5^{(Q)}$ satisfy

$$\begin{aligned} &x^5-b_1'x^4+(b_2'-5)x^3+(4b_1'-b_3')x^2 \\ &+(5-3b_2'+b_4')x+(2b_3'-2b_1'-b_5')=0, \end{aligned} \quad (8)$$

where

$$\begin{cases} b_1'=a_1-a_{10}+1, \\ b_2'=a_2-a_9+b_1', \\ b_3'=a_3-a_8+b_2', \\ b'4=a_4-a_7+b_3', \\ b_5'=a_5-a_6+b_4'. \end{cases} \quad (9)$$

The LSP parameters are equivalent to the roots of (7) and (8). Notice that $0<\theta_1^{(P)}<\theta_1^{(Q)}<\cdots<\theta_5^{(P)}<\theta_5^{(Q)}<\pi$, hence

$$-2<d_1^{(P)}<d_1^{(Q)}<\cdots<d_5^{(P)}<d_5^{(Q)}<2. \quad (10)$$

This point will be useful in the root-solving procedure.

Since the roots of a 5-degree polynomial can not be calculated in a closed form, numerical algorithm is employed in this paper. The roots of (7) are firstly considered. If $(2b_3-2b_1-b_5)=0$, (7) has a root at $x=0$, then the other four roots are the zeros of a 4-degree polynomial, and can be calculated in a closed form. If $(2b_3-2b_1-b_5)\neq 0$, the inflexions of the left member of (7) are calculated. These inflexions, denoted by $t_1<t_2<t_3<t_4$, divide the interval $(-2,2)$ into five subintervals, each of which contains a root. Using the method of bisection, search three roots of (7) separately in the shortest three subintervals. Then the other two roots can be directly calculated and the LSP parameters associated with $P_{10}(z)$ have been obtained.

Considering the roots of (8), if $x=0$ is a root, the other four roots can be obtained by Cardano's formula. Otherwise, (10) ensures each of the five intervals $(d_1^{(P)},d_2^{(P)})$, $\cdots$, $(d_4^{(P)},d_5^{(P)})$, $(d_5^{(P)},2)$ contains a root of (8). The roots can thus be computed by a similar procedure as in dealing with $P_{10}(z)$.

4. ROOT-SOLVING ALGORITHM

Based on the above analysis, the root-solving algorithm is proposed as follows:

Step 0: Input $a_1\sim a_{10}$.

Step 1: Calculate $b_1\sim b_5$ in (4) and $b_1'\sim b_5'$ in (9).

Step 2: If $(2b_3-2b_1-b_5)=0$, then $x_1=0$. Calculate directly the four nonzero roots $x_2\sim x_5$ by Cardano's formulas. Jump to Step 7.

Step 3: Calculate the inflexions $t_1<t_2<t_3<t_4$ of the left member of (7).

Step 4: Select the shortest three intervals among $(-2,t_1)$, (t_1,t_2), (t_2,t_3), (t_3,t_4), $(t_4,2)$.

Step 5: Using the method of bisection, search the three roots $x_1,\ x_2,\ x_3$ of (7) separately in the intervals selected in Step 4.

Step 6: Calculate the other two zeros of (7) by

$$\begin{aligned} x_{4,5} &= \tfrac{1}{2}(b_1-U) \\ &\pm \sqrt{\tfrac{1}{4}(b_1+3U)(b_1-U)+V-b_2+5}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} U &= x_1+x_2+x_3, \\ V &= x_1x_2+x_1x_3+x_2x_3. \end{aligned}$$

Step 7: Order the zeros $x_1\sim x_5$ and obtain $d_1^{(P)}\sim d_5^{(P)}$.

Step 8: If $(2b_3'-2b_1'-b_5')=0$, then $y_1=0$. Calculate directly the four nonzero roots $y_2\sim y_5$. Jump to Step 12.

Step 9: Select the shortest three intervals among $(d_1^{(P)}$, $d_2^{(P)})$, $(d_2^{(P)}$, $d_3^{(P)})$, $(d_3^{(P)}$, $d_4^{(P)})$, $(d_4^{(P)}$, $d_5^{(P)})$, $(d_5^{(P)}$, $2)$.

Step 10: Using the method of bisection, search the roots $y_1,\ y_2,\ y_3$ of (8) separately in the intervals selected in Step 9.

Step 11: Calculate the other two zeros of (8) by

$$\begin{aligned} y_{4,5} &= \tfrac{1}{2}(b'_1 - \hat{U}) \\ &\pm \sqrt{\tfrac{1}{4}(b'_1 + 3\hat{U})(b'_1 - \hat{U}) + \hat{V} - b'_2 + 5}, \end{aligned} \tag{12}$$

where

$$\begin{aligned} \hat{U} &= y_1 + y_2 + y_3, \\ \hat{V} &= y_1 y_2 + y_1 y_3 + y_2 y_3. \end{aligned}$$

Step 12: Order $y_1 \sim y_5$ and obtain $d_1^{(Q)} \sim d_5^{(Q)}$.

Step 13: Output the LSP parameters in x-domain, $\{ \frac{-d_1^{(P)}}{2}, \frac{-d_1^{(Q)}}{2}, \ldots, \frac{-d_5^{(P)}}{2}, \frac{-d_5^{(Q)}}{2} \}$.

5. COMPUTATIONAL COMPLEXITY

The above algorithm concentrates on searching roots by the method of bisection and solving 4-degree algebra equation. Of particular note is that the optimized root-solving procedure of a 4-degree polynomial involves 20 multiplications, 34 add/sub, 2 divisions and 5 quare roots [3].

Kabal [4] suggests the maximum absolute difference of LSP parameters in x-domain to be 0.02. In the above algorithm, the difference occurred in bisection searching will be accumulated according to the formulas in Step 6 and Step 11. In order to lower the difference of LSP parameters, high precision Δ is needed in bisection searching. When Δ is set to be 0.006, the maximum absolute of difference of LSP parameters in x-domain is 0.0083.

The algorithm proposed in this paper is compared with the methods in [3] and [4]. Set the precision of bisection to $\Delta = 0.006$, the required number of operations is shown in Table 1.

Table 1 Computational complexity of three LSP calculation methods

	$\times$	$\div$	$+$	$\sqrt{}$
AD-LSP	254	4	334	7
Mixed-LSP	280	10	664	5
Kabal's method	620	10	1390	0

6. CONCLUSION

A new algorithm for computing the LSP parameters from 10th-order LPC coefficients is developed. The algebra decomposition technique is employed to deduce the computation of LSP parameters into root-solving procedures of two 5-degree algebra equations.

7. REFERENCES

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